CONTINUING EDUCATION Mathematics Department - MCV4U PRACTICE EXAM

PART A: Fill in the blanks. Write your answer on the line provided.

1. Evaluate $\lim\limits_{x \to 2} \left(\dfrac{\sqrt{x+2}-2}{x-2} \right)$ _____

2. Determine the $\lim\limits_{x\to\infty} \dfrac{3x^2+1}{4x^2-2}$ _____

3. What does $\lim\limits_{h\to 0} \dfrac{(x+h)^2 - x^2}{h}$ equal? _____

4. Does $f(x) = 2x^2 - 8x$ have any inflection points? _____

5. For the function $f(x) = \begin{cases} x^2+3, & x \geq 2 \\ 2x+3, & -1 \leq x < 2 \\ x^3, & x < -1 \end{cases}$

   (a) Evaluate $f(-1)$ _____

   (b) Evaluate $\lim\limits_{x\to -1} f(x)$ _____

   (c) Evaluate $\lim\limits_{x\to 2} f(x)$ _____

6 Differentiate

   (a) $f(x) = 2x^3 - x$ _____

   (b) $y = \dfrac{1}{x^4}$ _____

   (c) $f(x) = 5^{(2x)}$ _____

   (d) $f(x) = e^{\cos x}$ _____

   (e) $f(x) = \tan(3x-4)$ _____

7. Determine $g(f(2))$ if $f(x) = \sqrt{x^2+5}$ and $g(x) = x^2 + x - 2$ _____

8. If $y = u^3 + u^2 - 1$, where $u = \dfrac{1}{1-x}$, determine $\dfrac{dy}{dx}$ at x = 2. _____

9. Let $f(x) = \dfrac{2x^3 - x^2 + 2x + 1}{x^2+1}$. What is the **<u>equation</u>** of the oblique asymptote. _____

Part B: **Short Answer Questions**
Answer in the space provided

1. *Using First Principles* (ie: the limit definition of a derivative), determine the derivative of $f(x)=\frac{3}{x^2}$

2. Differentiate the following functions. **Do Not Simplify**.

a) $y=\frac{1+x^2}{1+x^3}$

b) $g(x)=\left(x^3-2x+3\right)^4$

c) $f(x)=\left(x^2+1\right)^3(7x+2)^2$

3. Find the equation of the line tangent to the curve USING IMPLICIT DIFFERENTIATION on $x^2 - 2xy + 8 = 0$ at the point (2,3) in ***standard form***.

4. Determine the limit of the following function $\lim_{x \to 125} \dfrac{125 - x}{x^{\frac{1}{3}} - 5}$

5. A train travels along a straight track in a way that its position, in meters, can be expressed as a function $s(t) = 6t^2 + 4t$ where t is in seconds.

a) Determine the average velocity between t = 3 s. and t = 6 s.
b) Determine the instantaneous velocity when t = 6 s.
c) Determine the acceleration.

Part C: Problem Solving

Show all your work, providing neat and clear solutions in the space provided.

1. Find and classify any turning points for the function $f(t) = 2t^3 - 21t^2 + 60t$

2. A conical paper cup, with radius of 5 cm and height of 15 cm, is leaking water at a rate of 2 cm^3/min. At what rate is the water level decreasing when the water is 3 cm deep? [$V = \frac{1}{3}\pi r^2 h$]

3. A can is to be made to hold a litre of oil. What dimensions of the can are needed to minimize the cost of the metal to make the can. [1 L = 1000 cm^3 , $V = \pi r^2 h$, $SA = 2\pi r^2 + 2\pi rh$]

4. The curve defined by $f(x) = \frac{x^2 + 2x - 3}{x^2}$ has $f^1(x) = \frac{-2x + 6}{x^3}$, and $f^{11}(x) = \frac{4x - 18}{x^4}$.

   a) State the equations of any asymptotes. Examine the end behaviours.

b) Determine the co-ordinates of the x and y intercepts.

c) Determine the coordinates of all maximum and minimum points. Justify your answers.

d) Determine the ***x coordinates*** of all points of inflection.

e) Graph the function and ***identify any significant features***.